The use of PbO to measure the magnitude of local electric fields.

- A brief history of the beam resonance techniques
  - The Stern Gerlach experiment
  - The Rabi resonance experiment
  - The Ramsey resonance experiment
- The simple physics of PbO
- What a PbO beam resonance probe may look like

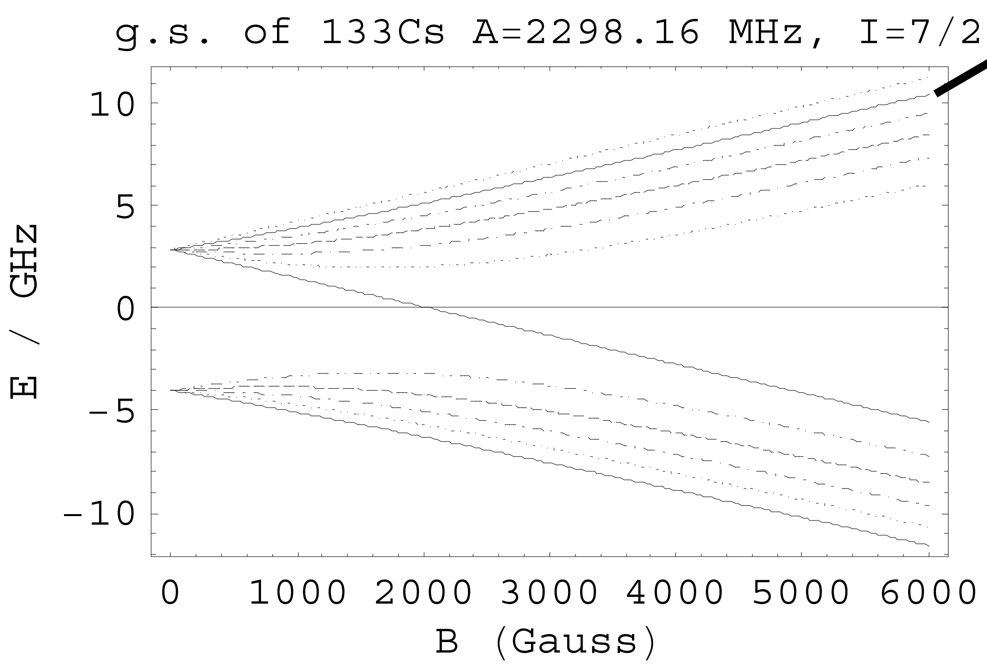
A brief history of the beam resonance techniques I: The Stern Gerlach experiment (1921)

Key Physics:

Quantum mechanics simplifies the motion of neutral particles in a field – quantized energy implies adiabatic motion.

Consider Zeeman (magnetic field dependent energies) of the quantum states of an atom:

$U_i(B)$

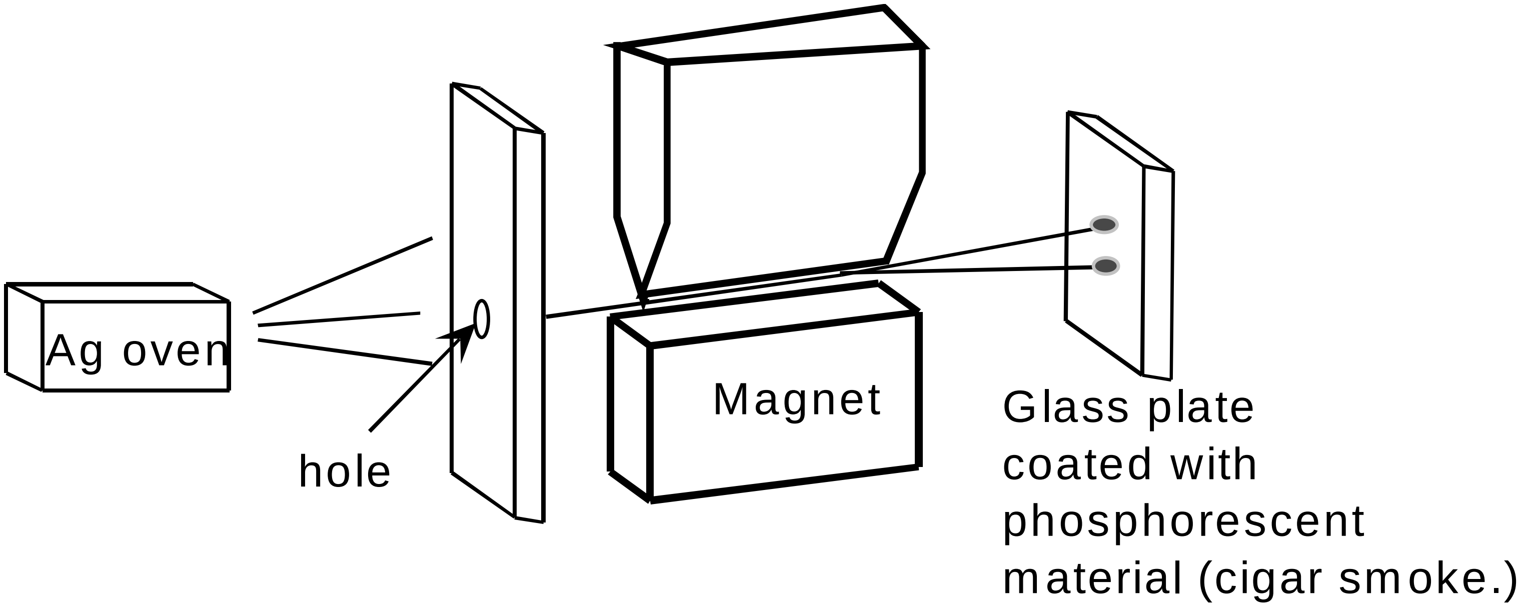


$$\vec{F}_i = -\vec{\nabla} U_i[B(\vec{r})]$$

The force depends only on the magnitude of the magnetic field and quantum state. (As the particle moves through the field, the quantization axis follows the field direction.)

A brief history of the beam resonance techniques I: The Stern Gerlach experiment (1921)

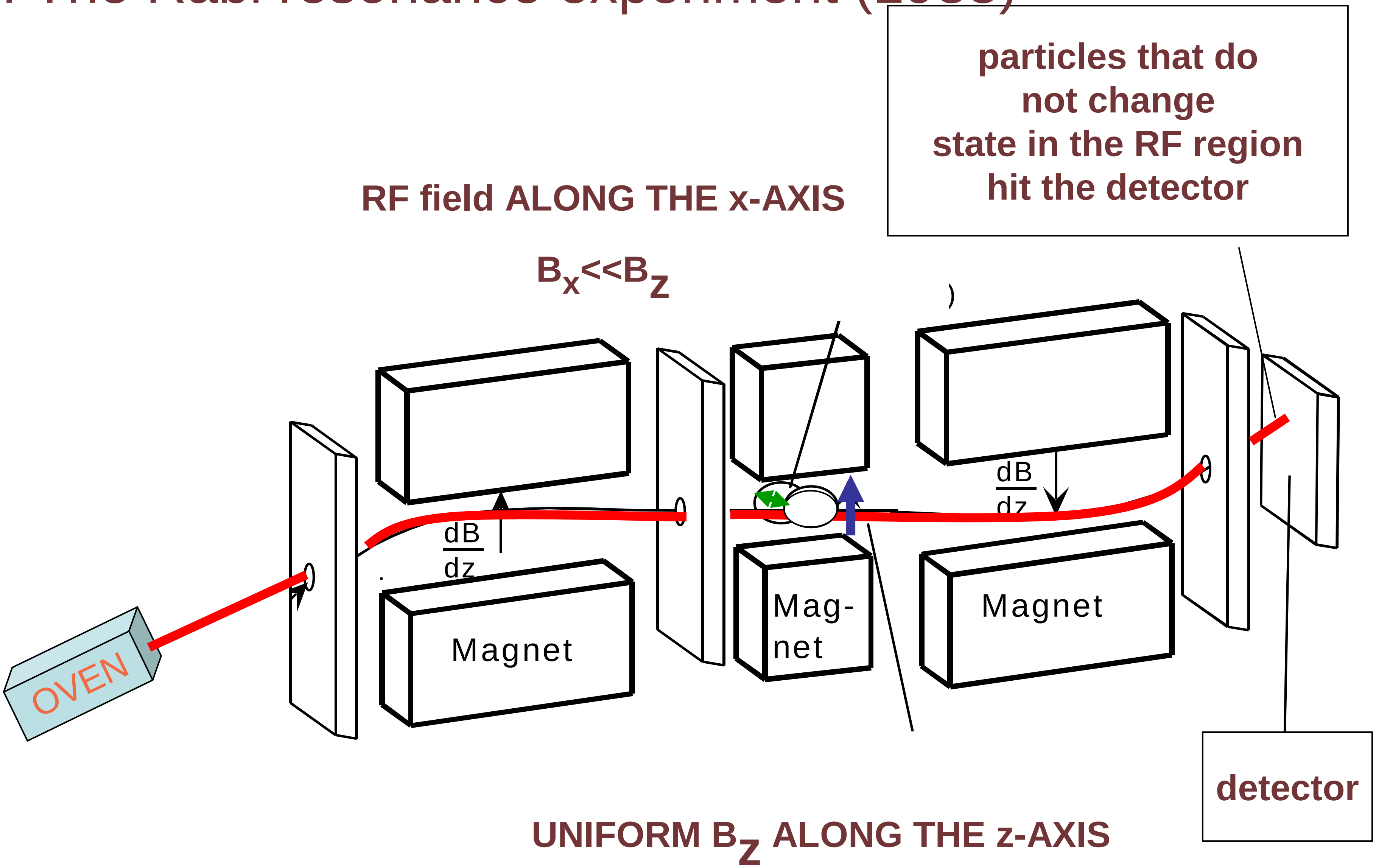
Each quantum state has a unique trajectory:



schematic of the famous measurement of the ½-integer angular momentum of silver

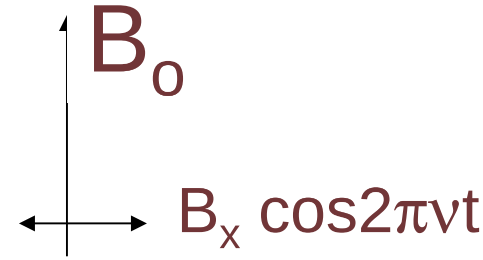
PROBLEM: very hard to quantify $U_i(B)$.

A brief history of the beam resonance techniques
II: The Rabi resonance experiment (1938)



The Rabi experiment can often described by considering two-levels with $P_1=|c_1|^2$ and $P_2=|c_2|^2$:

Schrödinger's Equation



$$\frac{i}{2\pi}\frac{d}{dt}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = H\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$H = \begin{bmatrix} \frac{\nu_o}{2} & \nu_\varepsilon \cos 2\pi\nu t \\ \nu_\varepsilon^* \cos 2\pi\nu t & -\frac{\nu_o}{2} \end{bmatrix} \approx \begin{bmatrix} \frac{\nu_o}{2} & \frac{\nu_\varepsilon}{2} e^{i2\pi\nu t} \\ \frac{\nu_\varepsilon^*}{2} e^{-i2\pi\nu t} & -\frac{\nu_o}{2} \end{bmatrix}$$

$\nu_o =$ $\nu_1 - \nu_2$ = energy difference between states

$\nu_\varepsilon \cos 2\pi\nu t = \langle\phi_2|H_1|\phi_1\rangle =$ perturbation provided by oscillating field.

Example: The linear Zeeman effect:

$\nu_o = g\,\mu_B B_o\,\Delta m,\quad \nu_\varepsilon = g\,\mu_B\,B_x\,\Delta m/2$

Using the rotating wave approximation, one finds the solution for c_1 and c_2 in terms of the initial state $(\cos\theta, e^{i\phi}\sin\theta)$:

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \cos(\pi\bar{\nu}t)\begin{pmatrix} \cos\theta \\ e^{i\phi}\sin\theta \end{pmatrix} +$$

$$+ i\sin(\pi\bar{\nu}t)\left[\frac{\Delta\nu}{\sqrt{\Delta\nu^2+\nu_\varepsilon^2}}\begin{pmatrix} \cos\theta \\ -e^{i\phi}\sin\theta \end{pmatrix} - \frac{1}{\sqrt{\Delta\nu^2+\nu_\varepsilon^2}}\begin{pmatrix} \nu_\varepsilon e^{i\phi}\sin\theta \\ \nu_\varepsilon^*\cos\theta \end{pmatrix}\right]$$

Here $\Delta\nu = \nu - \nu_o$ = detuning

and $\bar{\nu} = \sqrt{\Delta\nu^2 + \nu_\varepsilon^2}$ = the Rabi frequency

---

the populations are constant if $\Delta\nu >> \nu_\varepsilon$

the populations oscillate at $\bar{\nu}$ if $\Delta\nu=0$

Expected signal from a Rabi experiment

1. Atoms enter rf region with $P_{1,i} = 1$

2. An atom with velocity v exits rf region in state determined by $P_{1,f} = |c_1|^2$ evaluated at $t_f = L/v$

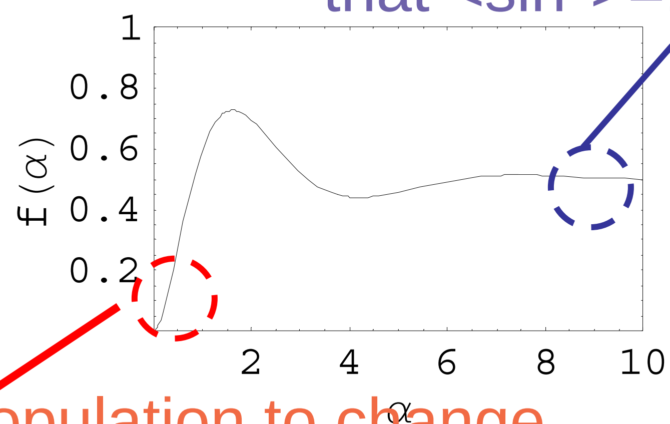
$$= 1 - \frac{\nu_\varepsilon^2}{\Delta\nu^2 + \nu_\varepsilon^2} \sin^2 \pi\bar{\nu}\, t_f \; = 1 - \frac{\nu_\varepsilon^2}{\Delta\nu^2 + \nu_\varepsilon^2} \sin^2 \pi(\Delta\nu^2 + \nu_\varepsilon^2)^{1/2} t_f$$

3. Signal is the average value $<P1_f>$ over the beam speed distribution

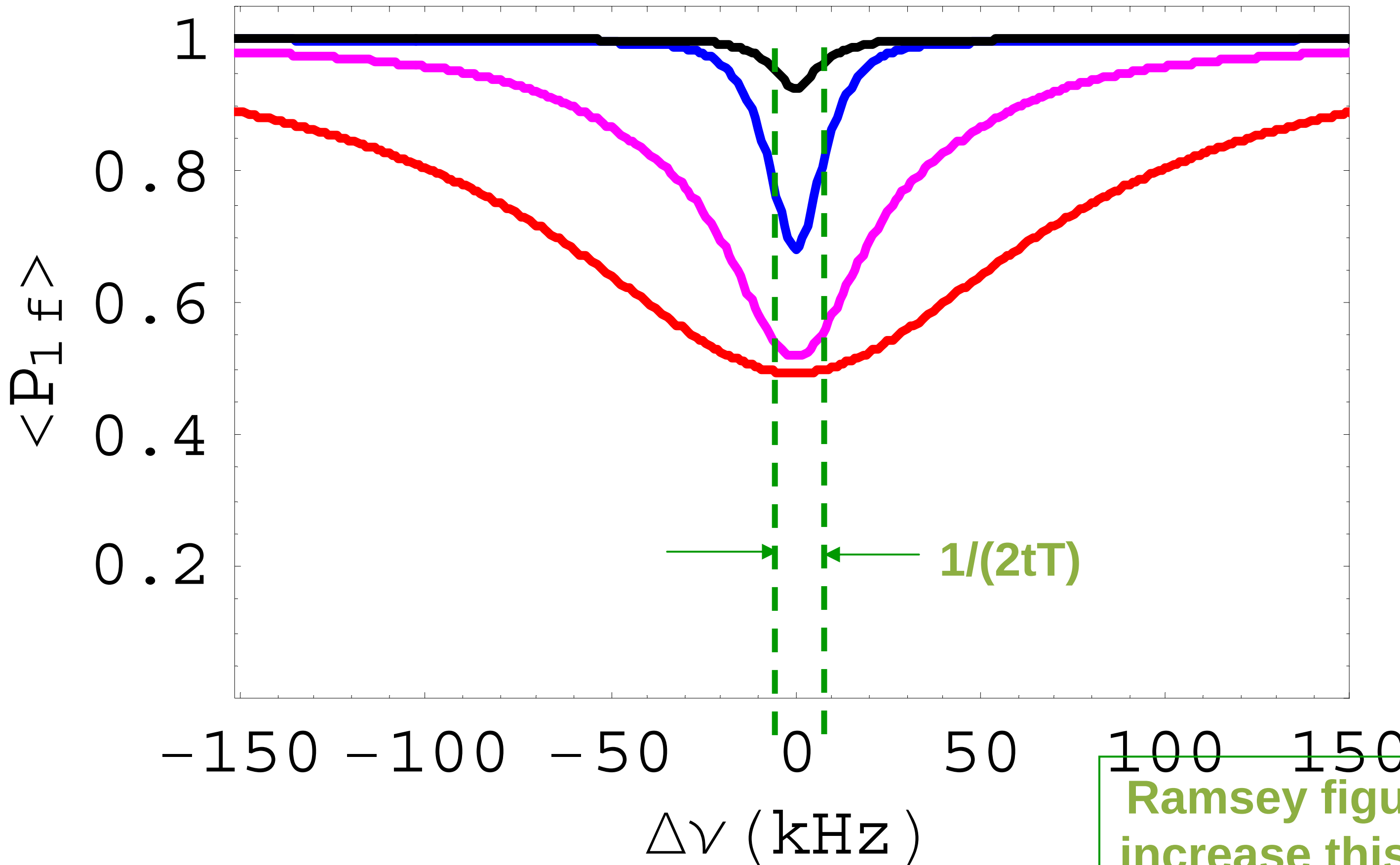
$$Sig = 1 - \frac{\nu_\varepsilon^2}{\Delta\nu^2 + \nu_\varepsilon^2} \int P(V) \sin^2\left(\frac{\pi\bar{\nu}L}{V}\right) dV$$

$$= 1 - \frac{\nu_\varepsilon^2}{\Delta\nu^2 + \nu_\varepsilon^2} f\left(\pi\bar{\nu} \frac{L}{\sqrt{2kT/m}}\right)$$

In rf region for so long that $<\sin^2>=1/2$.



Not in rf region long enough for population to change.



Rf intensity

ν_ε=2 kHz

ν_ε=5 kHz

ν_ε=30 kHz

ν_ε=80 kHz

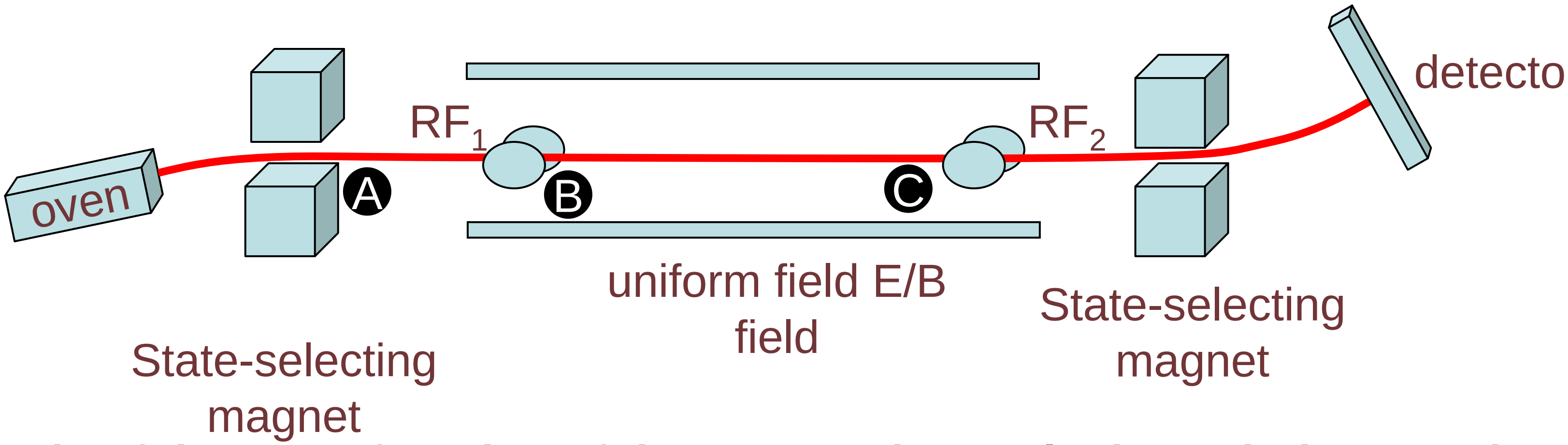
Ramsey figures out a way to increase this length to many meters.

Expected Rabi resonance signal for

$\nu_T = (2kT/m)^{1/2} / (2L) = 14\text{kHz}$

(m=100 amu , T=500K, and rf length L= 10mm)

A brief history of the beam resonance techniques II: The Ramsey resonance experiment (1949)

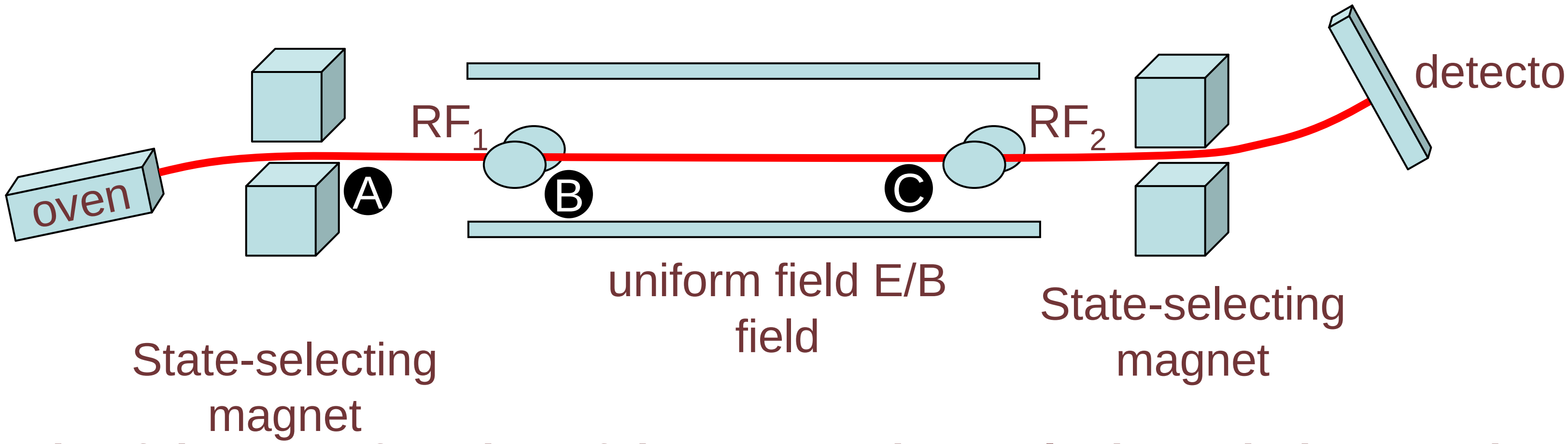


Analysis of the wave function of the atom as it travels through the experiment:

Ⓐ **Atoms exit magnets with**

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

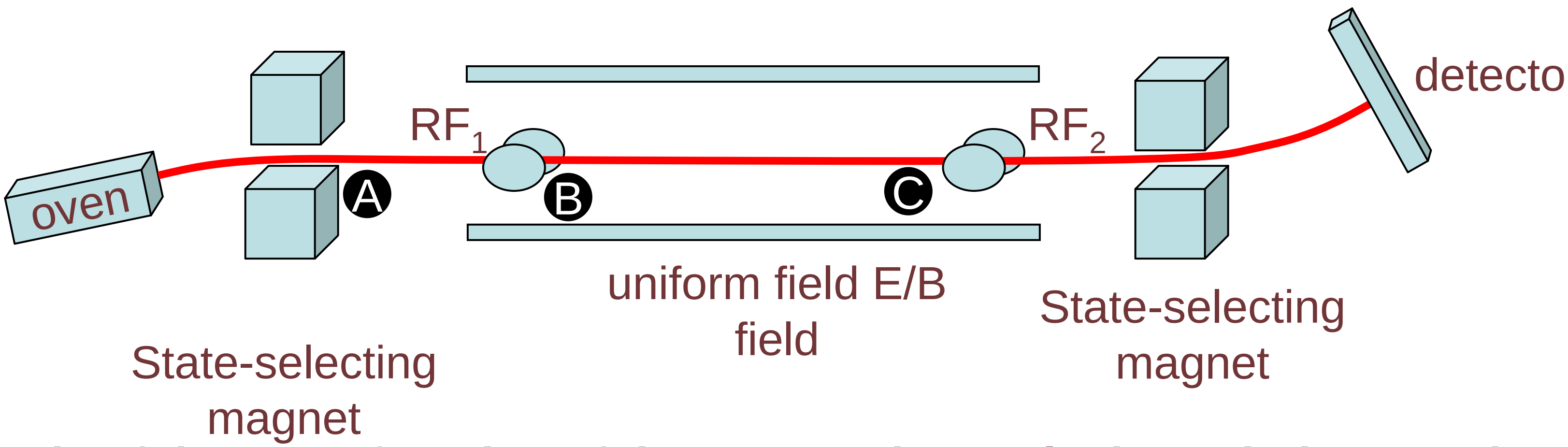
A brief history of the beam resonance techniques
II: The Ramsey resonance experiment (1949)



Analysis of the wave function of the atom as it travels through the experiment:

A $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

A brief history of the beam resonance techniques II: The Ramsey resonance experiment (1949)



Analysis of the wave function of the atom as it travels through the experiment:

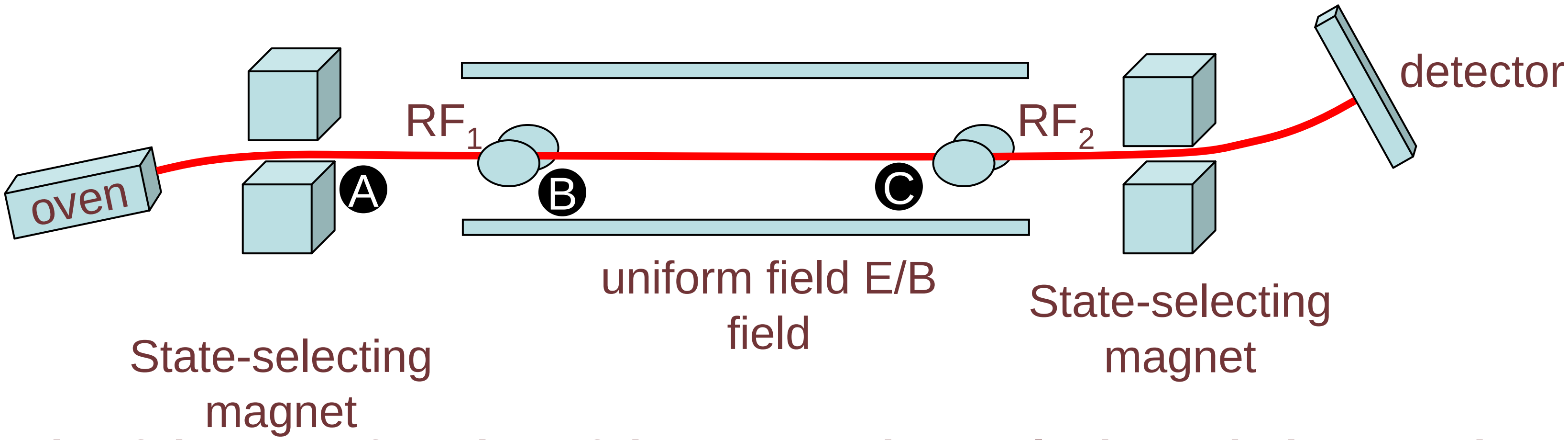
A $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

B In the rf region 1, we apply the previous result

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \approx \cos(\pi\bar{\nu}t)\begin{pmatrix} \cos\theta \\ e^{i\phi}\sin\theta \end{pmatrix} + i\sin(\pi\bar{\nu}t)\left[\frac{\Delta\nu}{\sqrt{\Delta\nu^2+\nu_\varepsilon^2}}\begin{pmatrix} \cos\theta \\ -e^{i\phi}\sin\theta \end{pmatrix} - \frac{1}{\sqrt{\Delta\nu^2+\nu_\varepsilon^2}}\begin{pmatrix} \nu_\varepsilon e^{i\phi}\sin\theta \\ \nu_\varepsilon^*\cos\theta \end{pmatrix}\right]$$

with θ=0 (so c_1=1) and assuming $\Delta\nu << \nu_\varepsilon,\ \nu_\varepsilon \in \mathrm{Re},\ \nu_\varepsilon > 0,$

A brief history of the beam resonance techniques II: The Ramsey resonance experiment (1949)



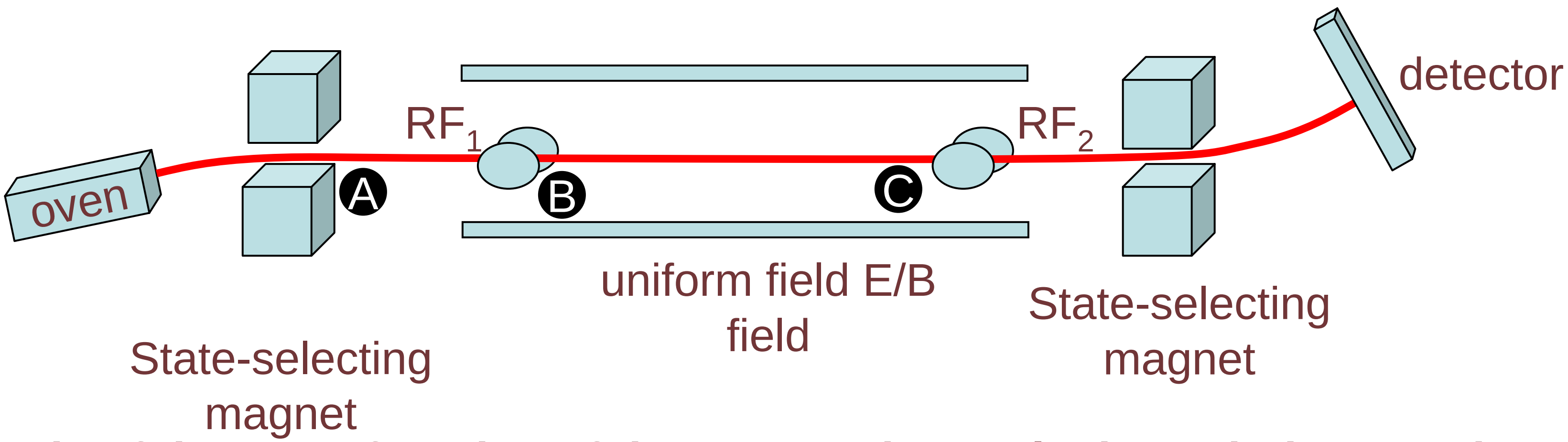
Analysis of the wave function of the atom as it travels through the experiment:

Ⓐ $$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Ⓑ In the rf region 1, we apply the previous result

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \cos \pi \nu_\varepsilon t_{Rf1} \\ -i \sin \pi \nu_\varepsilon t_{Rf1} \end{pmatrix}$$

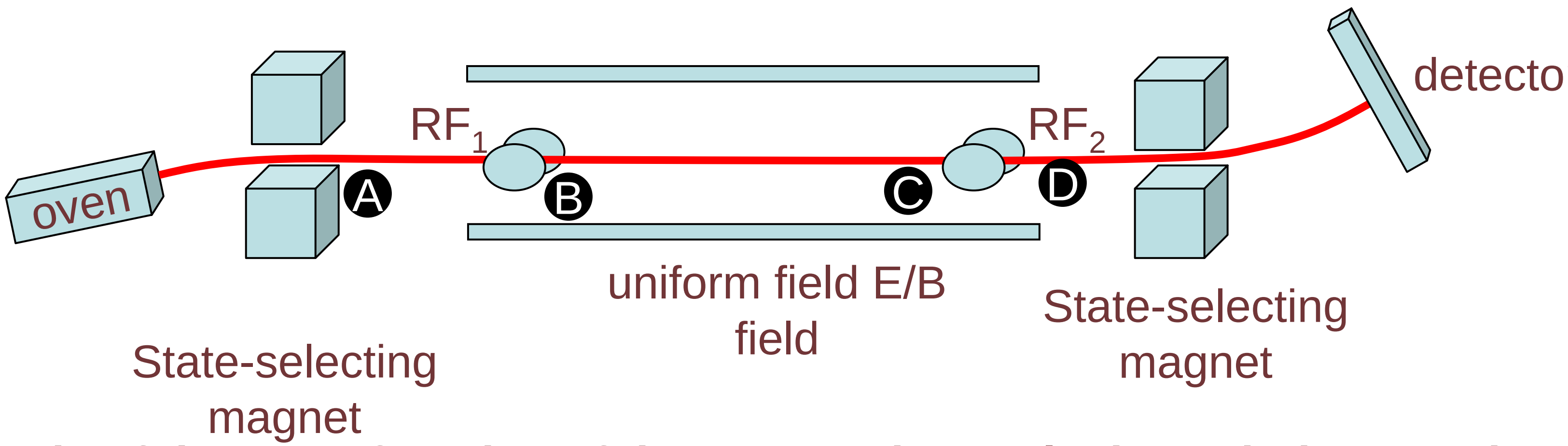
A brief history of the beam resonance techniques II: The Ramsey resonance experiment (1949)



Analysis of the wave function of the atom as it travels through the experiment:

A $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ B $\begin{pmatrix} \cos \pi \nu_{\varepsilon} t_{Rf1} \\ -i \sin \pi \nu_{\varepsilon} t_{Rf1} \end{pmatrix}$

A brief history of the beam resonance techniques
II: The Ramsey resonance experiment (1949)



Analysis of the wave function of the atom as it travels through the experiment:

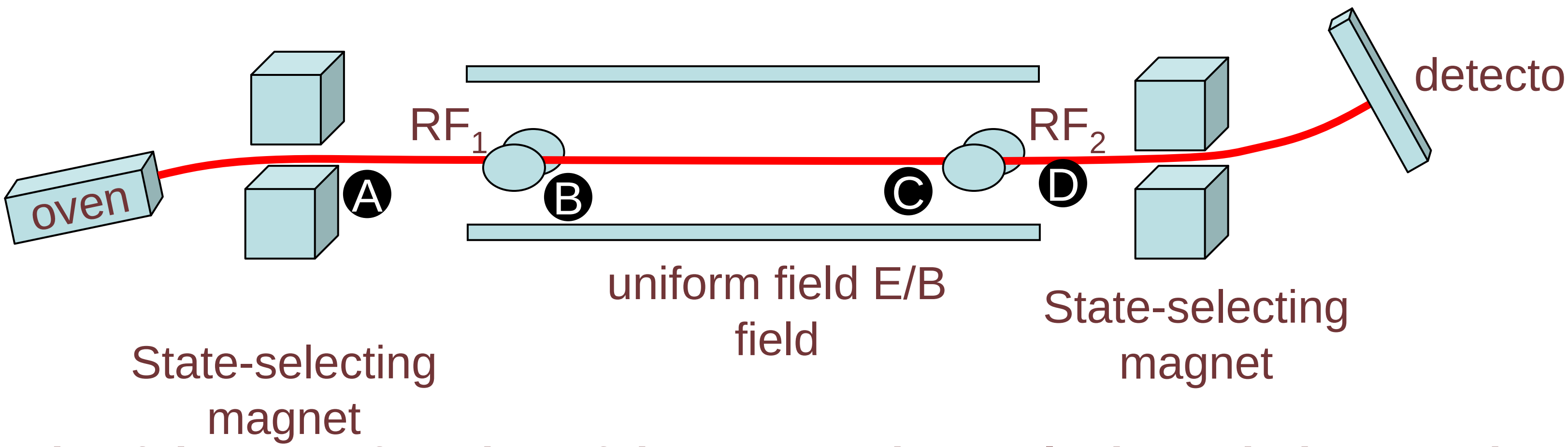
A $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ B $\begin{pmatrix} \cos \pi\nu_\varepsilon t_{Rf1} \\ -i \sin \pi\nu_\varepsilon t_{Rf1} \end{pmatrix}$ C $\begin{pmatrix} \cos \pi\nu_\varepsilon t_{Rf1} \\ e^{-i(2\pi\nu_o t_d + \pi/2)} \sin \pi\nu_\varepsilon t_{Rf1} \end{pmatrix}$

C In the drift region, the two states accumulate an additional pha

this phase matters!

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} e^{i\pi\nu_o t} \cos \pi\nu_\varepsilon t_{Rf1} \\ -ie^{-i\pi\nu_o t} \sin \pi\nu_\varepsilon t_{Rf1} \end{pmatrix} = e^{i\,arb} \begin{pmatrix} \cos \pi\nu_\varepsilon t_{Rf1} \\ e^{-i(2\pi\nu_o t_d + \pi/2)} \sin \pi\nu_\varepsilon t_{Rf1} \end{pmatrix}$$

A brief history of the beam resonance techniques II: The Ramsey resonance experiment (1949)



Analysis of the wave function of the atom as it travels through the experiment:

A $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ B $\begin{pmatrix} \cos \pi \nu_\varepsilon t_{Rf1} \\ -i \sin \pi \nu_\varepsilon t_{Rf1} \end{pmatrix}$ C $\begin{pmatrix} \cos \pi \nu_\varepsilon t_{Rf1} \\ e^{-i(2\pi\nu_o t_d + \pi/2)} \sin \pi \nu_\varepsilon t_{Rf1} \end{pmatrix}$

D Recall the two-level Hamiltonian:

$$H = \begin{bmatrix} \frac{\nu_o}{2} & \frac{\nu_\varepsilon}{2} e^{i2\pi\nu t} \\ \frac{\nu_\varepsilon^*}{2} e^{-i2\pi\nu t} & -\frac{\nu_o}{2} \end{bmatrix}$$

We see if $t \to t + t_d$ our solution for (c_1, c_2) applies with

$$\nu_\varepsilon \to \nu_\varepsilon e^{i2\pi\nu t_d}$$

Start with

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \cos(\pi\bar{\nu}t)\begin{pmatrix}\cos\theta \\ e^{i\phi}\sin\theta\end{pmatrix} + i\sin(\pi\bar{\nu}t)\left[\frac{\Delta\nu}{\sqrt{\Delta\nu^2+\nu_\varepsilon^2}}\begin{pmatrix}\cos\theta\\ -e^{i\phi}\sin\theta\end{pmatrix} - \frac{1}{\sqrt{\Delta\nu^2+\nu_\varepsilon^2}}\begin{pmatrix}\nu_\varepsilon e^{i\phi}\sin\theta \\ \nu_\varepsilon^*\cos\theta\end{pmatrix}\right]$$

let

$$\theta \to \pi\nu_\varepsilon t_{Rf}$$

$$\phi \to -(2\pi\nu_o t_d + \pi/2)$$

$$\nu_\varepsilon \to \nu_\varepsilon e^{i2\pi\nu t_d}$$

and assuming $\Delta\nu \ll \nu_\varepsilon,\ \nu_\varepsilon \in \mathrm{Re},\ \nu_\varepsilon > 0, t_{Rf1} = t_{Rf2} = t_{Rf}$

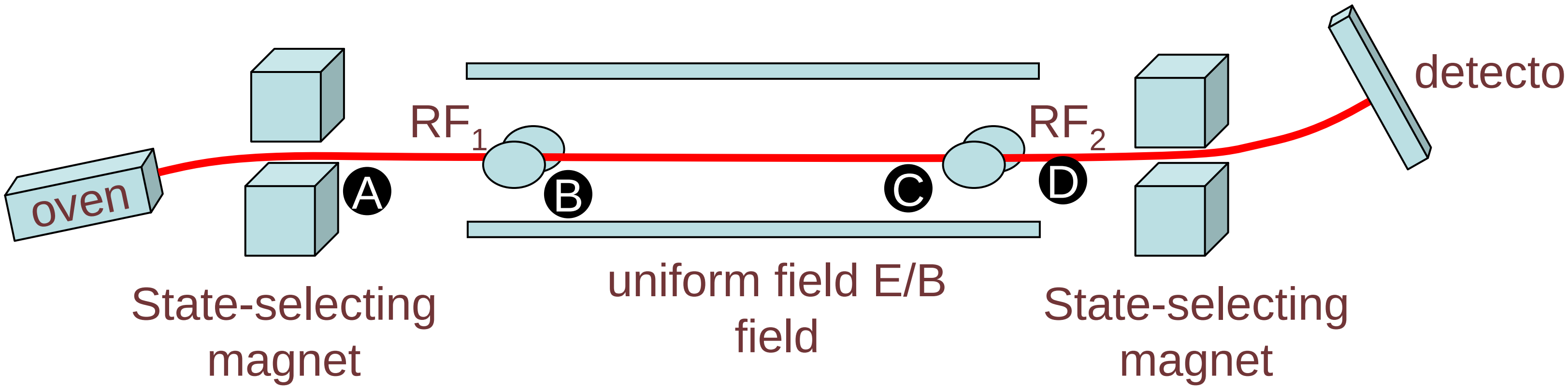
D

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \cos^2 \pi\nu_\varepsilon t_{Rf} \\ \tfrac{1}{2} e^{-i(2\pi\nu_o t_d + \pi/2)} \sin 2\pi\nu_\varepsilon t_{Rf} \end{pmatrix} - i \begin{pmatrix} e^{i(2\pi(\nu-\nu_o)t_d - \pi/2)} \sin^2 \pi\nu_\varepsilon t_{Rf} \\ \tfrac{1}{2} e^{-i2\pi\nu t_d} \sin 2\pi\nu_\varepsilon t_{Rf} \end{pmatrix}$$

The final population P_1 is given by

$$P_{1atD} = 1 - \cos^2(\pi\Delta\nu t_d)\sin^2(2\pi\nu_\varepsilon t_{Rf})$$

A brief history of the beam resonance techniques II: The Ramsey resonance experiment (1949)



Analysis of the wave function of the atom as it travels through the experiment:

A

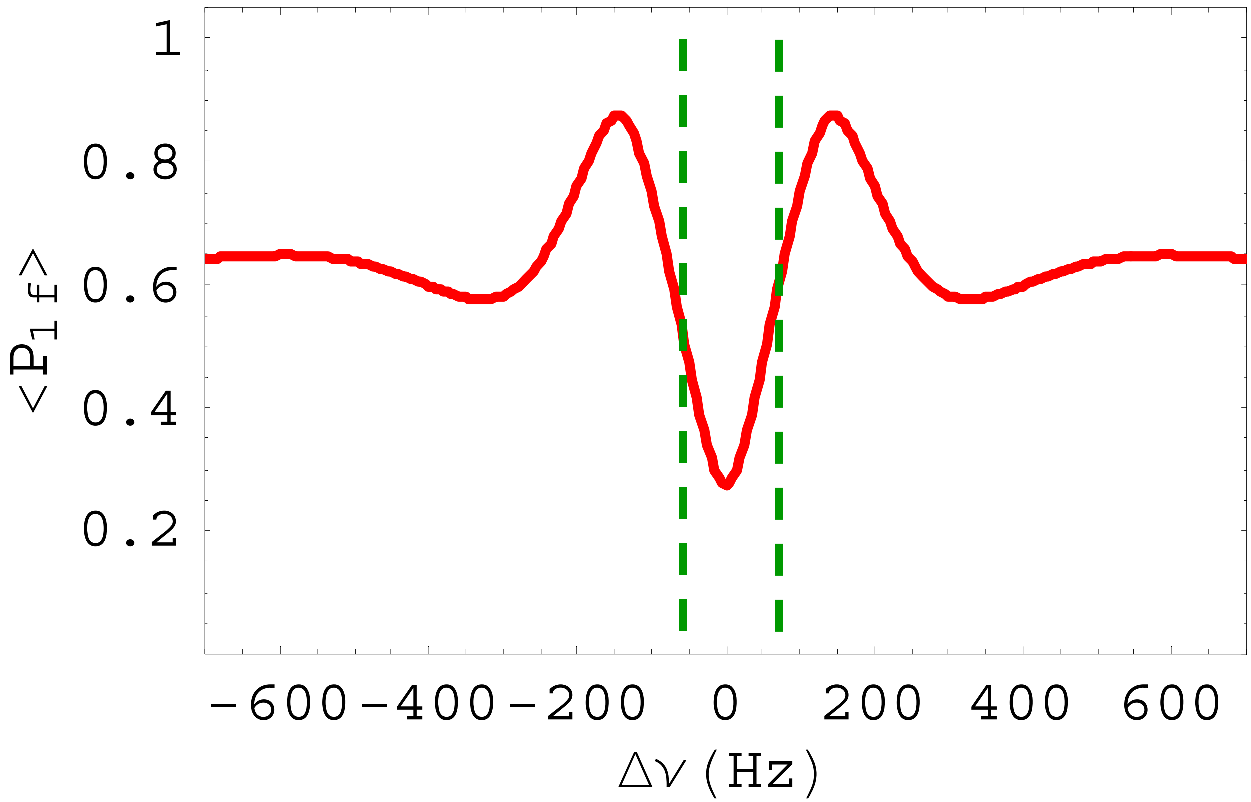
$P_1=1$

D

$$P_1 = 1 - \cos^2(\pi\,\Delta\nu\, t_d)\sin^2(2\pi\nu_\varepsilon t_{Rf})$$

Signal= $\langle P_1\rangle = \int P(V)\left[1-\cos^2(\pi\,\Delta\nu\frac{d}{V})\sin^2(2\pi\nu_\varepsilon\frac{d_{RF}}{V})\right]dV$

$$= \int P(V)\left[1-\cos^2(\pi\,\Delta\nu\frac{d}{V})\sin^2(\frac{\pi}{2}\frac{V_\varepsilon}{V})\right]dV$$

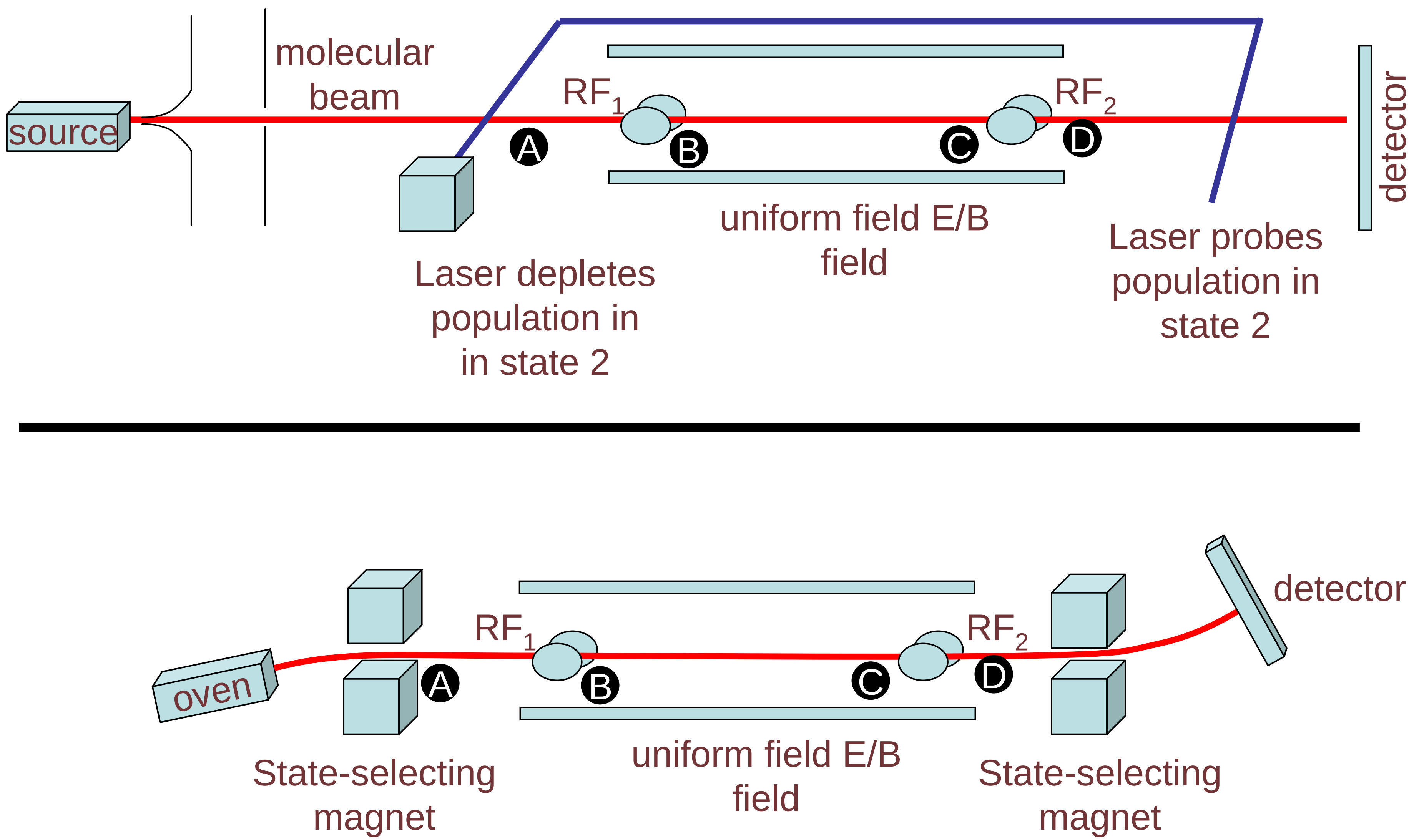


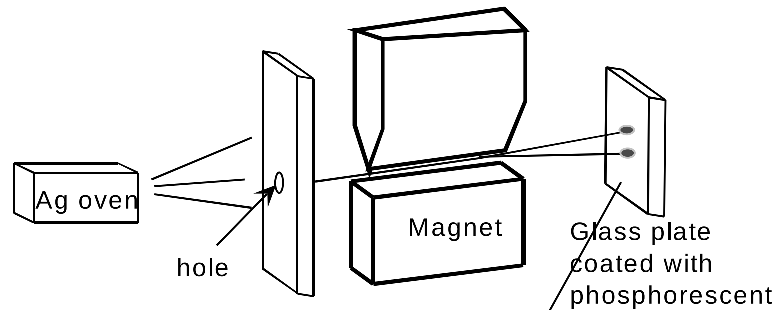
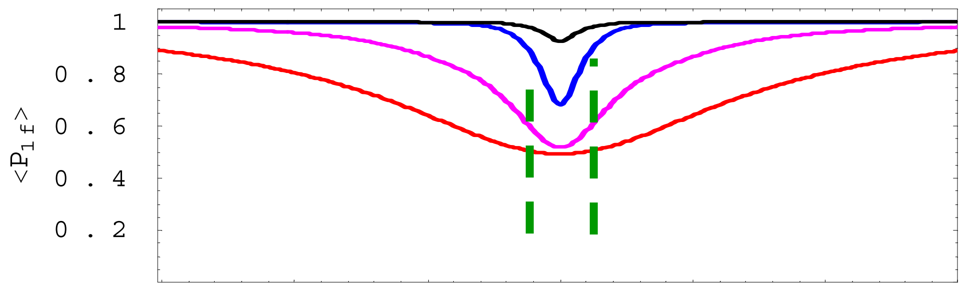
Expected Ramsey resonance signal for

$\nu_T = (2kT/m)^{1/2} / 2L = 140$ Hz

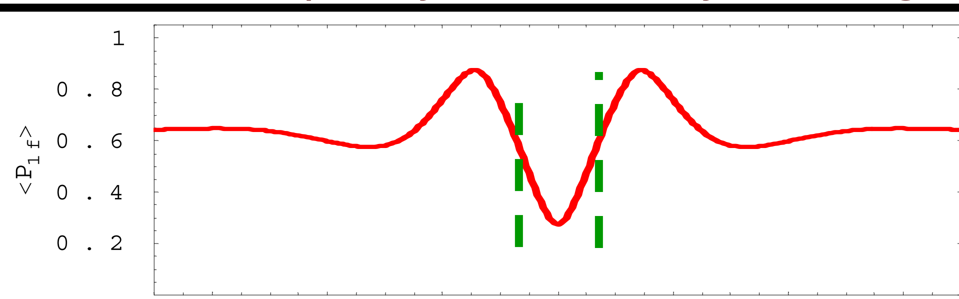
(m=100 amu , T=500K, and rf length L= 1000mm)

In a modern Ramsey experiments, the Stern-Gerlach magnets are often replaced by a laser to get a much bigger throughput.



Experiment	Measurement	Typical fwhm of resonance for 500K, 100amu
Stern Gerlach (1921)	) Final point of trajectory gives information of quantum state.	crude
Rabi (1938)		14KHz

signal as a function of frequency and intensity of a single RF perturbation

Ramsey (1949)		14OHz

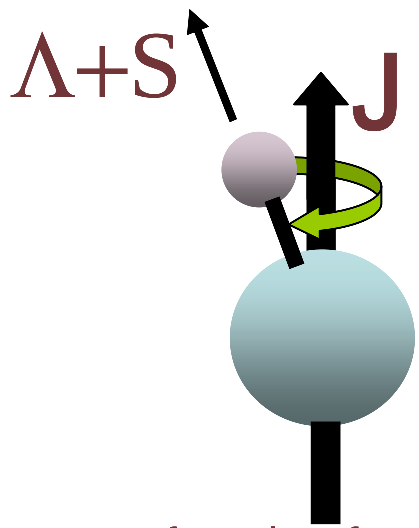
signal as a function of frequency and intensity of two phase-locked RF perturbations

The simple physics of of PbO.

1.The ground state of PbO is a $^1\Sigma$ state.

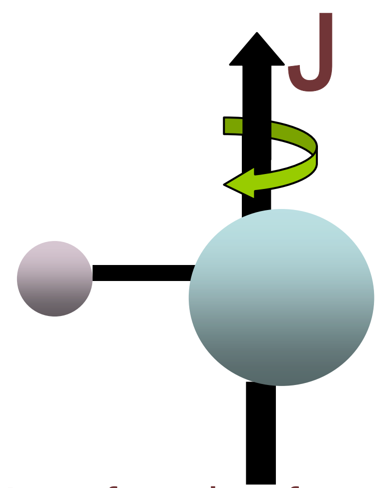
component of angular momentum on the nuclear axis Λ	term symbol
0	Σ
1	Π
2	Δ

In general, diatomic molecules move like symmetric tops



wave function for nuclear motion a Wigner rotation matrix: $D^J_{\Lambda m}$

With L = 0, the molecule is a simple rotor



Wave function for nuclear motion spherical harmonics Y_{JM}

The simple physics of of PbO.

2. The ground state of PbO interacts weakly with with a B field:

1	electronic angular momenta: $\mu_B\,(\mathbf{L}+g\mathbf{S})\cdot\mathbf{B}$	0 for $^1\Sigma$ states
m_e/M_p	nuclear angular momenta: $\mu_N\,\mathrm{I}\cdot\mathbf{B}$	0 because $I_{Pb}=I_O=0$
m_e/μ_{PbF}	nuclear angular momenta: $g_{mol}\,\mu_N\,\mathbf{J}\cdot\mathbf{B}$	$g_{PbF}= -0.1623$

The simple physics of of PbO.

3. The ground state of PbO has a large (4.64 Debye) dipole moment.

All together

$$H = \mu\vec{J} \bullet \vec{B} - D\hat{r} \bullet \vec{E} + \beta_{rot} J^2$$

1 volt/cm E-field perturbation bigger than that of 1 Tesla B field!

$\mu = -1.237$ MHz/Tesla

$D = 2.335$ MHz/(volt/cm)

$\beta_{rot} = 9293$ MHz

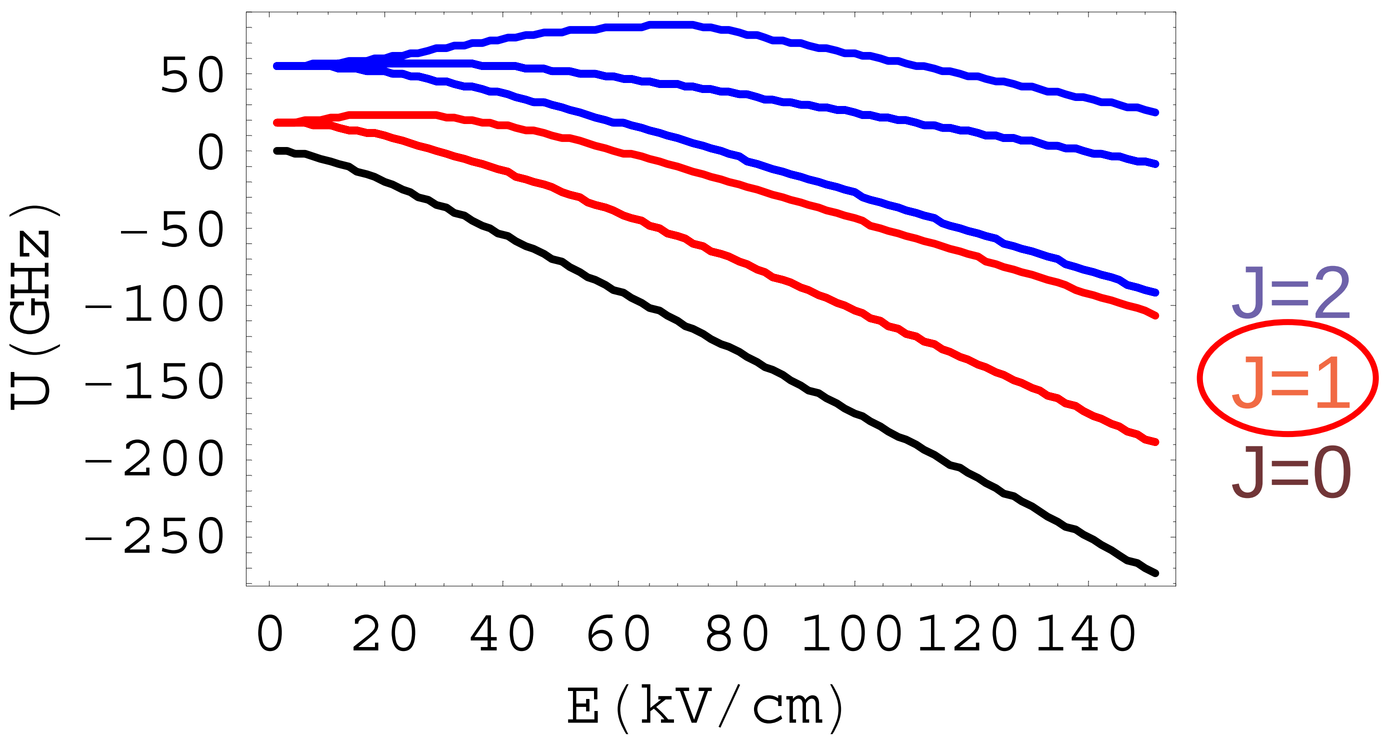
Basis set: Eigen functions of J^2 and J_z with

$$\left| J, M \right\rangle = Y_{JM}(\theta, \varphi)$$

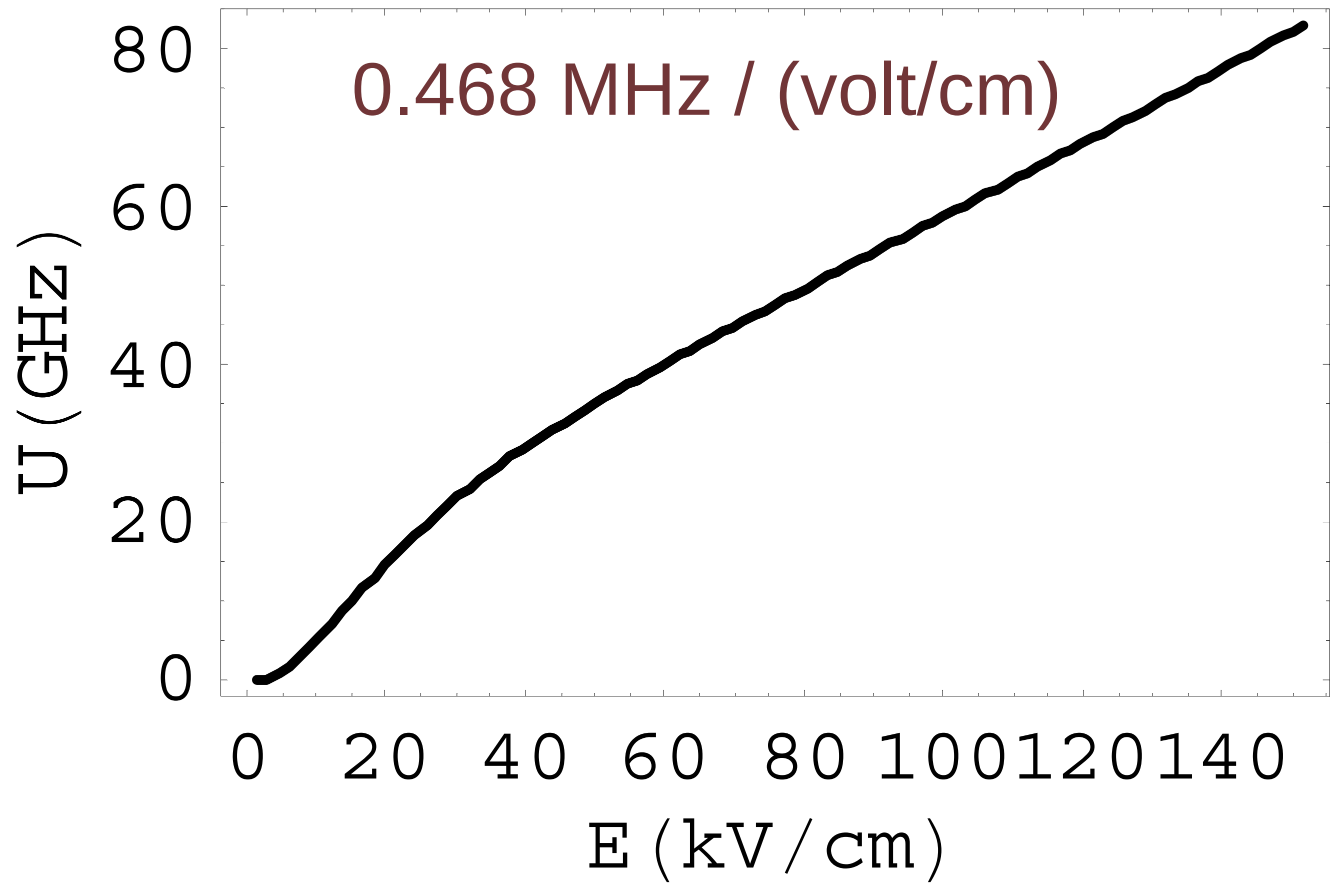
To find quantum states as a function of $\vec{E}$ *and* $\vec{B}$ *numerically determine the eigenvalues of*

$$\langle J'M'|H|JM\rangle =$$

$$\langle J'M'|\mu\vec{J}\bullet\vec{B} - D\hat{r}\bullet\vec{E} + \beta_{rot}J^2|JM\rangle$$

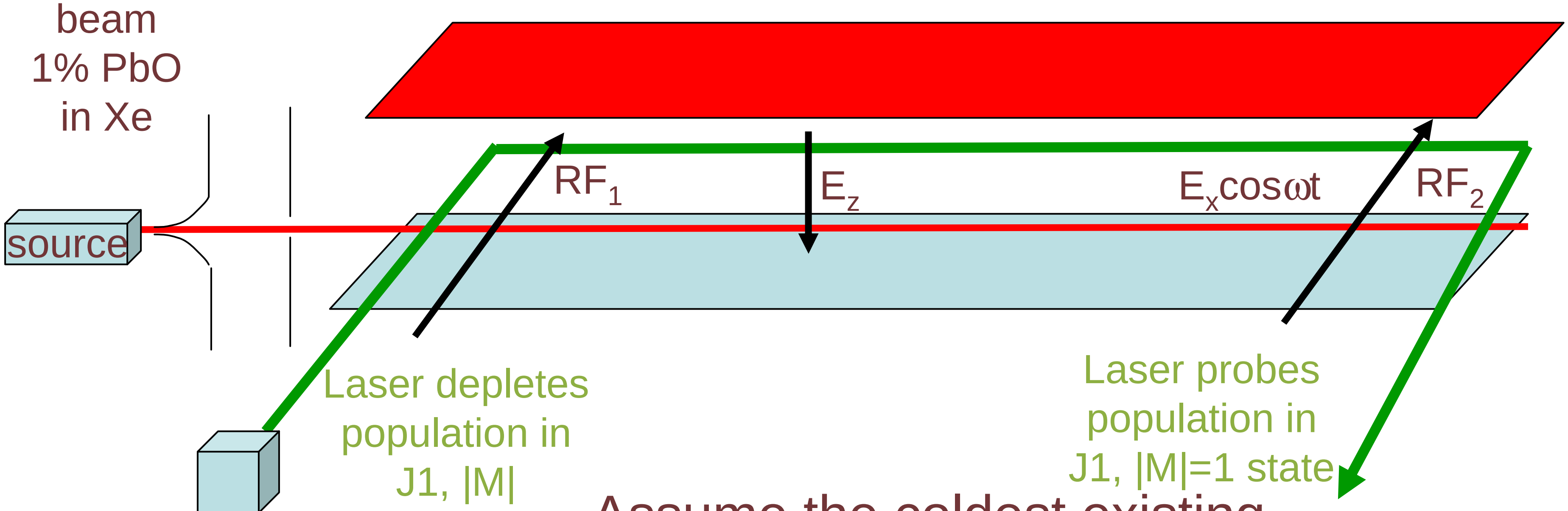


Energy difference between the J=1, |M|=1 and M=0 states of PbO as a function of electric field:



$$\text{Ramsey FWHM} = \frac{\text{volt / cm}}{2(\text{INTERACTION TIME})\ 0.468\ \mu s^{-1}}$$

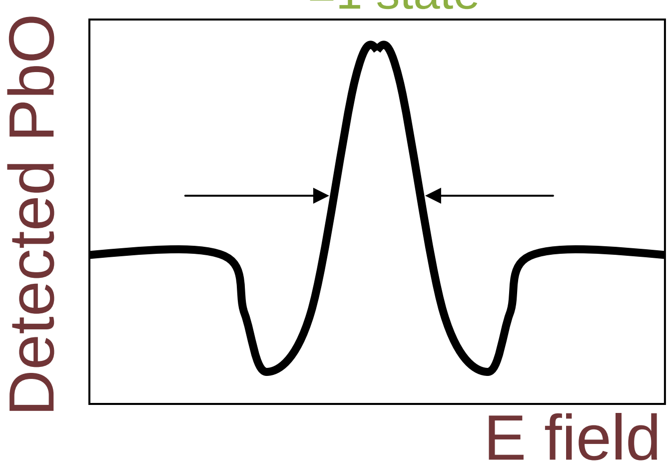
Ramsey probe of the E-field dependent J=1 |M|=0 and J=1 |M|=1 states of PbO



Assume the coldest existing molecular beam:

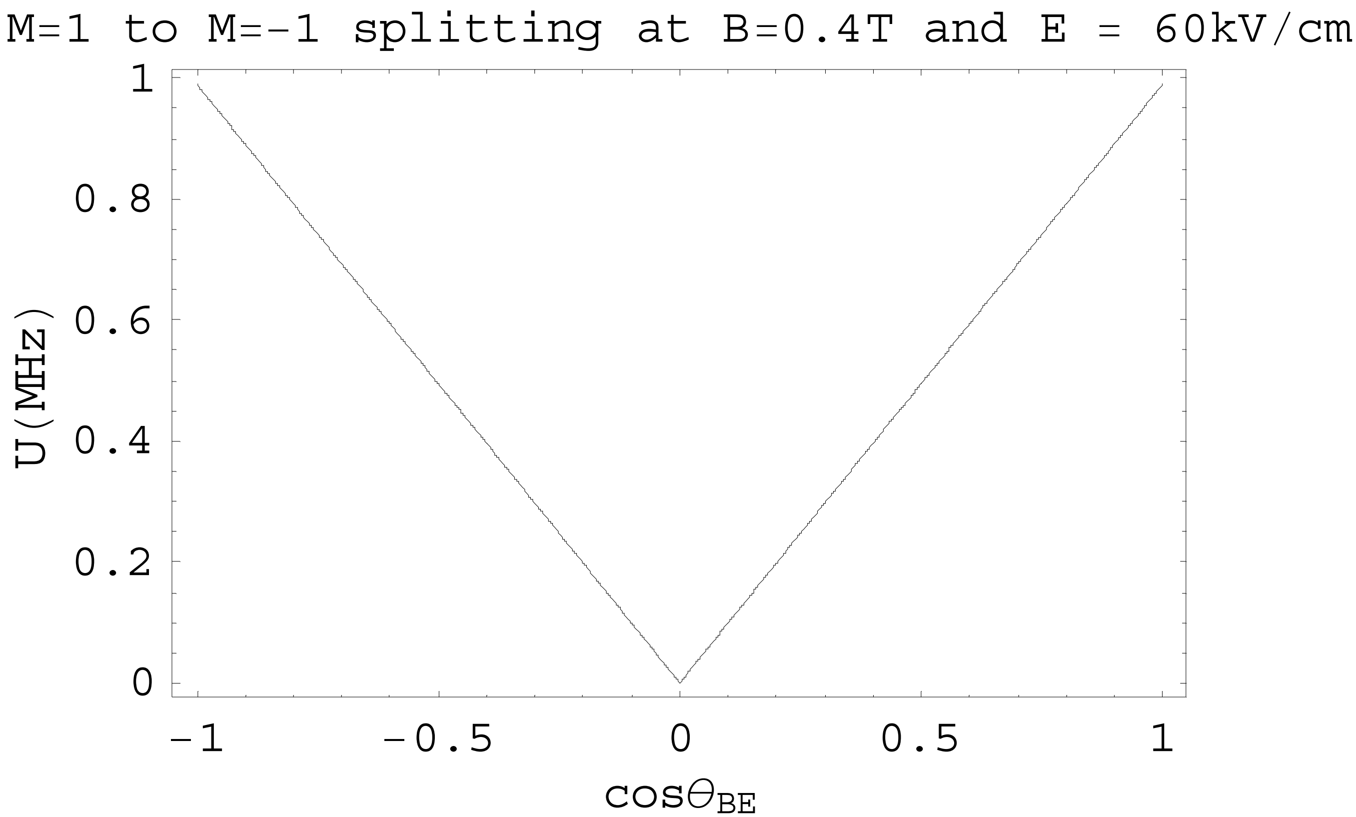
$v \approx 0.25$ mm/µs

4ms interaction time

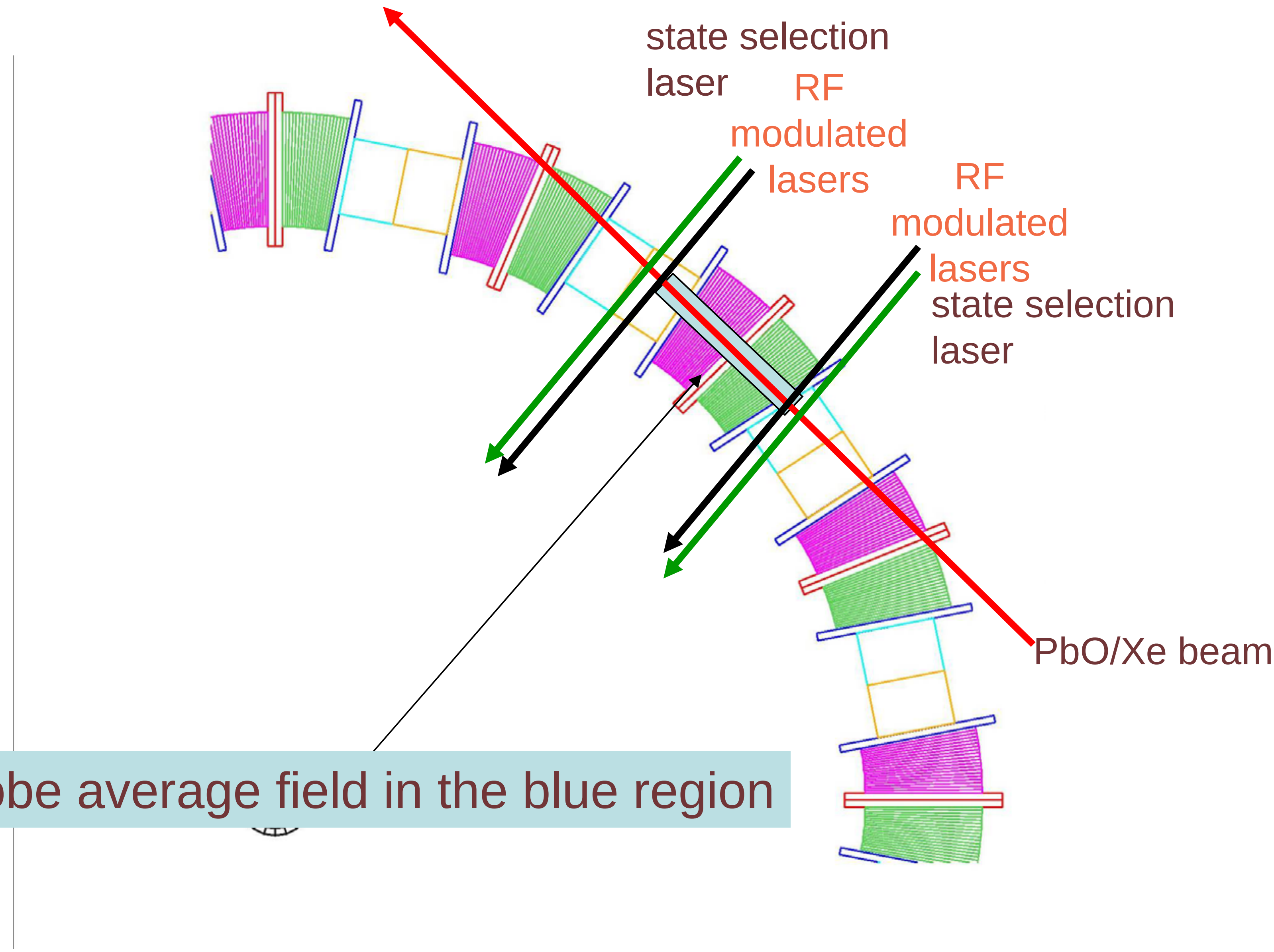


$$\text{fwhm} \approx \frac{260\ \mu\text{V/cm}}{\text{L/m}}$$

What about B-field dependence?



If we measure to 100Hz, $\cos\theta_{BE} = 0$ +/- $\mathbf{10^{-4}}$



Probe average field in the blue region

Expected Result

- Assumed Parameters:

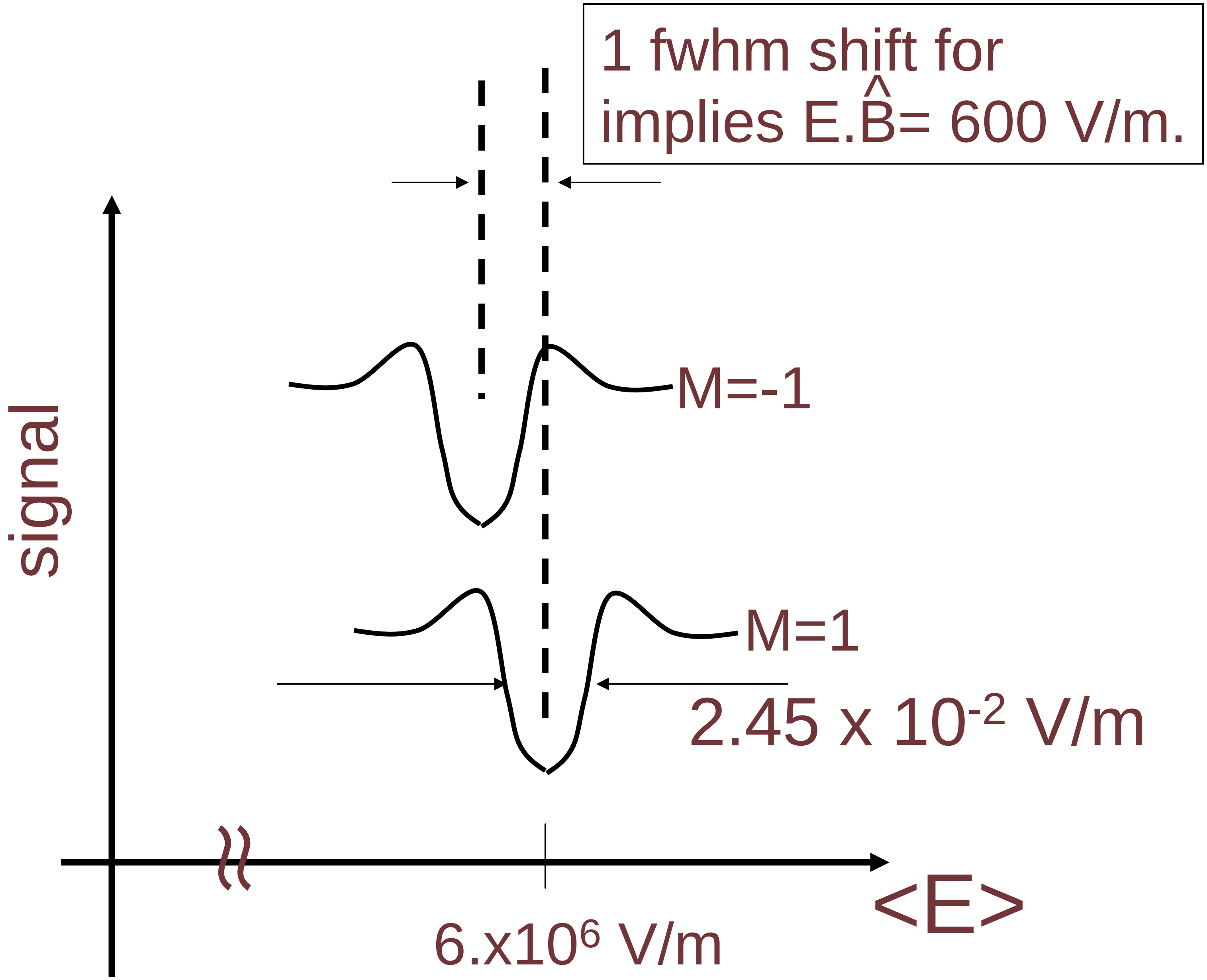
  E = 6 MV/m,   B=0.4 T

  laser RF frequency fixed (locked to an atomic clock) to 60. GHz.

  0.24 mm/μs atomic beam from PbO seeded in He cooled to 10K.

  1m long interaction region.

  ability to switch from (M=-1 to M=0) resonance to (M=1 to M=0) transition with laser polarization tricks.


1 fwhm shift for
implies E.$\hat{B}$= 600 V/m.
signal
M=-1
M=1
2.45 x 10^{-2} V/m
6.x10^{6} V/m
<E>